

Photon Distribution Amplitude in approach with nonlocal condensates ¹

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Abstract

We investigate magnetic susceptibility and the leading twist light-cone distribution amplitude for the real photon in the framework of the nonlocal condensate approach at LO accuracy of perturbative expansion.

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1 Introduction

The knowledge of the photon Distribution Amplitude (DA) and quark condensate Magnetic Susceptibility (MS) is quite important for hard exclusive processes involving photon emission. Examples include transition form factors like $\gamma^* \rightarrow \pi\gamma$ with one real and one virtual photon [1, 2], deeply-virtual Compton scattering (DVCS) [3], radiative hyperon and meson decays like $\Sigma^+ \rightarrow p\gamma$, $B \rightarrow \rho\gamma$, $B \rightarrow K^*\gamma$ and $D^* \rightarrow D\gamma$ [4, 5, 6, 7, 8, 9].

MS χ has been introduced in the pioneering work [10]: $\langle 0 | \bar{q} \sigma_{\mu\nu} q | 0 \rangle_F = e_q \chi \langle \bar{q}q \rangle F_{\alpha\beta}$. Here $\langle \bar{q}q \rangle$ is the quark condensate, $F_{\alpha\beta}$ the field-strength tensor of the external EM field, and the subscripts F indicates that the vacuum expectation value is taken in the vacuum in the presence of the field $F_{\alpha\beta}$. Different theoretical approaches have been used to get this quantity: QCD SR for proton and neutron magnetic moments [10], Borel SR analysis of two-points correlator [11, 12, 13], the correlator of vector and nonsinglet axial-vector currents [14, 15], instanton liquid model [17] and the instanton inspired model [15, 18].

Following [13, 9], we define the leading twist normalized photon DA $\phi_\gamma(x, \mu^2)$ using the matrix elements of the tensor current with light-like separations:

$$\langle 0 | \bar{q}(z) \sigma_{\mu\nu} \mathcal{C}(z, 0) q(0) | \gamma(q, \lambda) \rangle \Big|_{z^2=0} = i e_q \chi(q^2) \langle \bar{q}q \rangle (\epsilon_\mu(q, \lambda) q_\nu - \epsilon_\nu(q, \lambda) q_\mu) \int_0^1 dx e^{ix(zq)} \phi_\gamma(x, \mu^2).$$

The Wilson line [19] $\mathcal{C}(z, 0) = \mathcal{P} \exp [ig \int_0^z A_\mu(\tau) d\tau^\mu]$ is inserted in the matrix element for gauge-invariance. In the above definitions, $|\gamma(q, \lambda)\rangle$ is the one-photon state of momentum p and polarization vector $\epsilon_\mu(q, \lambda)$, and μ^2 is the normalization point. The parameter $\chi(q^2 \rightarrow 0)$ is the so-called magnetic susceptibility of the quark condensate.

The photon DA was introduced in [9], where closeness of the photon DA to asymptotic form $\phi_\gamma^{as}(x) = 6x(1-x)$ was claimed on the basis of the standard QCD SR. Nevertheless, in instanton model approaches [16, 17], the photon DA has a flat-like form and no end-point suppression. The

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authors of [13] showed the instability of this QCD SR. This instability allows the photon DA to have a non-asymptotic form in contrast to [9]. Here we develop a nonlocal condensate (NLC) approach [20, 21] to the MS and the photon DA that significantly improves the properties, and expand the region of applicability of QCD SR.

2 Nondiagonal correlator in NLC approach

By using the Background Field formalism [13] one can get an equivalent definition of the photon DA via a correlator of the nonlocal tensor current $q(0)\sigma_{\alpha\beta}\mathcal{C}(0,z)q(z)$ (on the light-cone) with the vector one, j_μ ,

$$\int d^4y e^{iqy} \langle 0 | T [\bar{q}(0)\sigma_{\alpha\beta}\mathcal{C}(0,z)q(z)j_\mu(y)] | 0 \rangle \Big|_{z^2=0} = i\chi(q^2) \langle \bar{q}q \rangle (q_\alpha g_{\beta\mu} - q_\beta g_{\alpha\mu}) \int_0^1 dx e^{ix(zq)} \phi_\gamma(x, \mu^2). \quad (1)$$

This nondiagonal correlator can be applied for extracting magnetic susceptibility and photon DA by using the operator product expansion (OPE) method. A remarkable property of this correlator is that the leading-order (LO) contribution is completely defined by the nonperturbative vacuum and receives no perturbative contributions in the chiral limit at all. The diagram corresponding to the LO contribution is shown in the right-hand side of Fig. 1. For calculation of this correlator we would like to apply the NLC technique. Here we need to introduce only nonlocal scalar condensate: $\langle \bar{q}(0)\mathcal{C}(0,x)q(x) \rangle = \langle \bar{q}q \rangle \int_0^\infty f_S(\nu) e^{\nu x^2/4} d\nu$ which is parameterized in the general case by the distribution functions $f_S(\alpha)$ of vacuum quarks in virtualities ν , [20, 21]. Explicit forms of these functions should be taken, generally speaking, from the theory of nonperturbative QCD vacuum that is still unknown. In the absence of such a theory we suggested ansatz that takes into account only the main effect of nonlocality, the non-zero average virtuality λ_q^2 of quarks in the QCD vacuum [22]: $f_S(\nu) = \delta(\nu - \lambda_q^2/2)$. The nonlocality parameter $\lambda_q^2/2 = \langle k^2 \rangle = \int_0^\infty f_S(\nu) \nu d\nu$ characterizes the average momentum of quarks in the QCD vacuum and has been estimated in QCD SRs [23, 24] and on the lattice [25, 26]: $\lambda_q^2 = 0.45 \pm 0.05 \text{ GeV}^2$. One can get a relation between the studied correlator at LO (in the r.h.s) and the magnetic susceptibility multiplied by the photon DA (in the l.h.s):

$$\chi(Q^2) \phi_\gamma(x; Q^2, \mu^2) = \begin{cases} x \int_0^\infty \frac{d\beta}{\beta} f_S(\beta) \exp\left(-\bar{x} \frac{Q^2}{\beta}\right) + (x \rightarrow \bar{x}), & \text{NLC-case} \\ \frac{\delta(x) + (x \rightarrow \bar{x})}{Q^2}, & \text{standard condensates } (\lambda_q^2 \rightarrow 0) \end{cases} \quad (2)$$

where $\bar{x} \equiv 1 - x$, $Q^2 = -q^2 > 0$. From (2) follows that $\chi(Q^2 \rightarrow 0)$ has a meaning for the NLC-case in contrast with the local limit one [13].

To tell the truth, applying the OPE at low $Q^2 \leq \lambda_q^2/2$ can rule out the basis of the approach. Nevertheless, in this bold extrapolation $Q^2 \rightarrow 0$ one has

$$\chi^{\text{LO}} = \int_0^\infty d\beta f_S(\beta)/\beta \xrightarrow{\delta\text{-ansatz}} 2/\lambda_q^2 = 4.5(0.5) \text{ GeV}^{-2} \quad (3)$$

that provides us a rough but reasonable estimate for δ -ansatz that agrees with the previous results: $\chi \approx 2.3 - 5.6 \text{ GeV}^{-2}$ [13, 15, 17, 14, 11, 12, 18], see Fig. 2 for details. The experimentally based

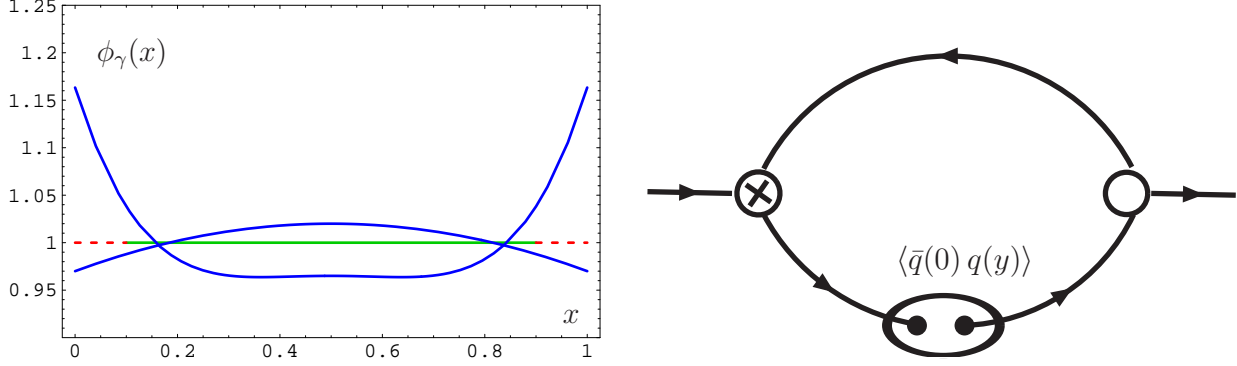


Figure 1: **Left panel:** Photon DA in different approaches. The displayed curves are: convex curve – instanton inspired model [16], concave curve – instanton liquid model [17], flat line – our LO result in NLC approach. **Right panel:** LO contribution to studied correlator (1).

constraints on this value $\chi \approx 2.4 - 3.6 \text{ GeV}^{-2}$ was obtained in [7, 8], where the asymptotic behavior of the photon DA was assumed. These constraints are shown on the left panel of Fig. 2.

The NLC expressions (2) can be used to obtain MS and photon DA at any virtuality Q^2 of photon. As $Q^2 \rightarrow 0$ limit we get from (2) a model-independent LO photon DA: $\phi_\gamma^{\text{LO}}(x) = \theta(1 > x > 0)$, it does not depend on the choice of $f_S(\beta)$ as opposed to MS. Let us emphasize that this derivation does not fix the ϕ_γ^{LO} behavior in the vicinity of the end-points $x = 0, 1$. This remains unclear due to unreliability of the OPE in this region. If one uses the $\phi_\gamma(1/2) = 1$ assumption, the experimentally based constraints [7, 8] on MS will be shifted to higher values $\chi \approx 3.5 - 5.4 \text{ GeV}^{-2}$, which is demonstrated on the right panel of Fig. 2.

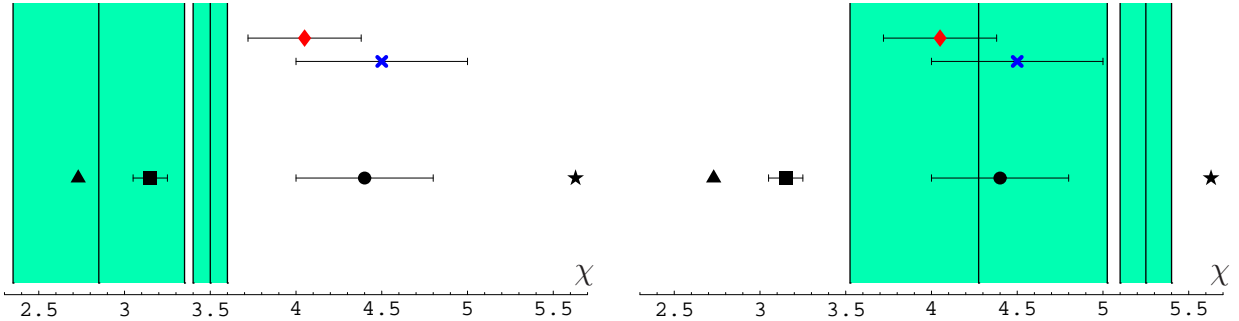


Figure 2: Theoretical constraints [13, 15, 11, 12, 14] on MS χ in comparison with the experimentally based constraints [7, 8]. The displayed constraints are: \blacklozenge – our phenomenological estimation (4) based on f_ρ and f_ρ^T data from [27], \times – our NLC LO result (3), \blacktriangle – instanton inspired model [15], \blacksquare – Borel QCD SR with local condensate [13], \bullet – QCD SR constraints [11, 12], \star – recent estimation based on AVV-correlator and pion dominance [14], shaded areas – experimentally based constraints [7, 8]. **Left panel:** Original estimations (shaded area) [7, 8] based on asymptotic DA assumption. **Right panel:** For flat-like DA experimentally based constraints (shaded areas) shifted to higher value of MS.

3 Magnetic susceptibility from spectral density

Let us consider an alternative way to get photon properties as an example of MS. We write the physical spectral density of this correlation function as a sum over several narrow resonances plus a smooth continuum starting at a threshold s_0 . Assuming quark-hadron duality, the continuum contribution can be represented by the perturbative imaginary part of the radiative correction [13]. As the first approximation one can retain the contribution of two lowest-lying ρ -meson states only

and use the same value of the continuum threshold as obtained in the ρ -meson sum rules for the correlation function of vector currents, $s_0 = 2.8 \text{ GeV}^2$ [27]. Thus, one gets magnetic susceptibility at $Q^2 = 0$ and $\mu^2 = 1 \text{ GeV}^2$:

$$\chi = -\frac{f_\rho f_\rho^T}{m_\rho \langle \bar{q}q \rangle} - \frac{f_{\rho'} f_{\rho'}^T}{m_{\rho'} \langle \bar{q}q \rangle} + \frac{8\alpha_s}{3\pi} \frac{1}{s_0} = 4.05(33) \text{ GeV}^{-2}, \quad (4)$$

$$\begin{aligned} \text{where } \langle \bar{q}q \rangle &= (-0.25 \text{ GeV})^3, & \alpha_s(\mu = 1 \text{ GeV}) &= 0.56, & s_0 &= 2.8 \text{ GeV}^2, \\ m_\rho &= 0.7755(4) \text{ GeV}, & f_\rho^L &= 0.201(5) \text{ GeV}^{-2}, & f_\rho^T &= 0.169(5) \text{ GeV}^{-2}, \\ m_{\rho'} &= 1.465(22) \text{ GeV}, & f_{\rho'}^L &= 0.175(10) \text{ GeV}^{-2}, & f_{\rho'}^T &= 0.140(10) \text{ GeV}^{-2}. \end{aligned}$$

Here we use decay constants that were got in [27] by the NLC Sum Rules approach. The estimate presented in (4) agrees well with the estimate in (3).

4 Conclusions

Let us summarize our findings.

(i) QCD SR approach with nonlocal condensates at LO accuracy allows us to easily get magnetic susceptibility as a function of photon virtuality Q^2 without singularity as $Q^2 \rightarrow 0$. The corresponding estimate is in good agreement with pervious estimations [11, 12, 13, 14, 15, 17, 7, 8, 18].

(ii) According to this approach, the photon DA $\phi_\gamma(x)$ has a flat-like form at medium $0 < x < 1$. This conclusion agrees with the instanton model methods [16, 17].

(iii) This kind of derivation does not fix the ϕ_γ^{LO} behavior in the vicinity of the end-points $x = 0, 1$.

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